Mixed strategies and best responses

For the following game, graph the best responses in mixed strategies and also find the Nash equilibrium in mixed strategies:

		Player 2	
		A	B
	X	-2,3	3,0
Player 1	Y	1,0	1,5

Solution

If we want to graph the best response functions, we should see when each player prefers to play a certain strategy based on the probability assigned by the other player. For example, for player 1 to play strategy X with a probability of 1, there are certain values for player 2's probabilities. Let's call player 2's probabilities (p,1-p). The payoff for player 1 when playing x given player 2's probabilities is:

$$U_1(x,(p,1-p)) = -2p + 3(1-p) = 3 - 5p \tag{1}$$

The payoff for player 1 when playing y given player 2's probabilities is:

$$U_1(y,(p,1-p)) = p + (1-p) = 1 \tag{2}$$

If we want to see when player 1 will play x, we propose that the utility of playing x is greater than playing y. $U_1(y,(p,1-p)) < U_1(x,(p,1-p))$. This gives us the result:

$$1 < 3 - 5p \text{ Therefore } p < 2/5 \tag{3}$$

If we call player 1's probabilities q and 1-q, the probability of playing the pure strategy x would be q=1. Therefore, we say that P1 plays q=1 when $p < 2/5$.

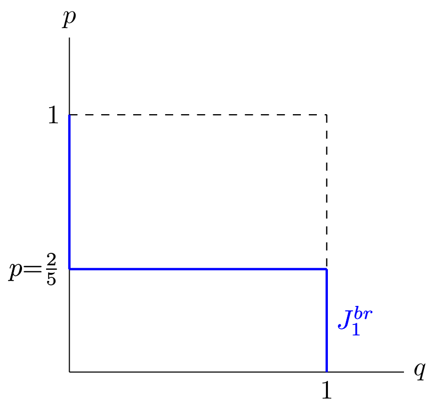

Figure 1: Best response functions

On the other hand, for player 1 to play the pure strategy y, it must be true that: $U_1(y,(p,1-p)) > U_1(x,(p,1-p))$

$$1 > 3 - 5p \text{ Therefore } p > 2/5 \tag{4}$$

So we say that $q = 0$ when $p > 2/5$. See Figure 1 that illustrates the best response of player 1 (blue line). Player 1 is only indifferent between the values of q when $p = 2/5$.

For player 2, the reasoning is similar; first, we calculate the expected utilities:

$$U_2((q,q-q),A) = 3q + 0(1-q) = 3q \tag{5}$$

$$U_2((q,1-q),B) = 0q + 5(1-q) = 5 - 5q$$

We propose that the utility of playing A is greater than playing B given the mixed strategy of player 1, $U_2((q,q-q),A) > U_2((q,1-q),B)$, then:

$$3q > 5 - 5q \text{ Therefore } q > 5/8 \tag{6}$$

We say then that player two will play $p = 1$, when $q > 5/8$. On the other hand, reversing the inequality, we get that the player will play $p = 0$ when $q < 5/8$. In graphical terms, in Figure 2, we see the best response of player 2 in the red line.

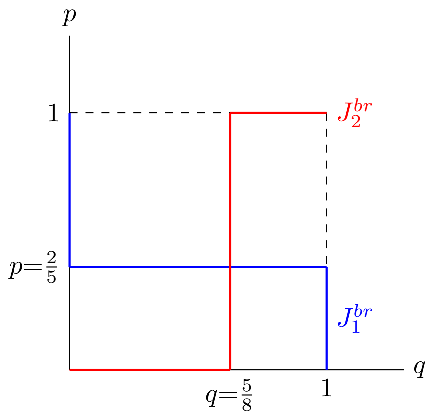

Figure 2: Best response functions